Parameters for Double Pendulum Model

Let us number the two pivots of the double pendulum 1 for the pivot that is stationary and 2 for the pivot that can move. Likewise, number the two rectangular pieces 1 for the piece attached to both pivots and 2 for the piece attached only to pivot 2. We can model the double pendulum, ignoring friction, with the following parameters:

- ℓ: distance between the two pivots
- m_1, m_2: masses of the two rotating pieces
- r_1, r_2: for each piece, distance from its pivot to its center of mass
- I_1, I_2: moment of inertia for each piece about its *pivot*

We will replace all but ℓ with the following dimensionless parameters, where k can be 1 or 2:

- $\rho = m_2/m_1$: mass ratio
- $\alpha_k = r_k/\ell$: relative distance to center of mass
- $\beta_k = I_k/(m_k\ell^2)$: moment of inertia normalized by the moment of a point mass at distance ℓ

These five parameters, along with the ratio g/ℓ, where g is gravitational acceleration, are used in our model implementation. The model is thus nondimensionalized, except for the time unit.

Model Equations

The state variables for the model are θ_1, θ_2, p_1, p_2, where θ_k is the angle of piece k from the downward vertical, and p_k is a generalized momentum.

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\begin{aligned}
\dot{\theta}_1 &= \frac{\rho\beta_2 p_1 - \rho\alpha_2\cos(\theta_1-\theta_2)p_2}{(\beta_1+\rho)\rho\beta_2 - \rho^2\alpha_2^2\cos^2(\theta_1-\theta_2)} \\
\dot{\theta}_2 &= \frac{(\beta_1+\rho)p_2 - \rho\alpha_2\cos(\theta_1-\theta_2)p_1}{(\beta_1+\rho)\rho\beta_2 - \rho^2\alpha_2^2\cos^2(\theta_1-\theta_2)} \\
\dot{p}_1 &= -\frac{g}{\ell}(\alpha_1+\rho)\sin\theta_1 - \rho\alpha_2\sin(\theta_1-\theta_2)\dot{\theta}_1\dot{\theta}_2 \\
\dot{p}_2 &= -\frac{g}{\ell}\rho\alpha_2\sin\theta_2 + \rho\alpha_2\sin(\theta_1-\theta_2)\dot{\theta}_1\dot{\theta}_2
\end{aligned}
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The model derivation is based on:

`http://instructor.physics.lsa.umich.edu/advlabs/Chaotic Double Pendulum/Pendulum_2010_04_12.pdf`